# Derivatives of Blocking Probabilities in Multirate Access Tree Networks

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***Abstract*—We consider a multirate access tree network accommodating service-classes of Poisson traffic, and focus on the determination of derivatives of Call Blocking Probabilities (CBP) with respect to offered traffic-load of any service-class. Based on the derivatives, we further show how we can successfully approximate CBP for small variations of offered traffic-load.**

***Index Terms*—Call Blocking, Derivatives, Poisson, Loss Model, Access Tree Network.**

## I. INTRODUCTION

THE determination of CBP derivatives not only has a theoretical value but also is of great importance in: a) network optimization in terms of capacity allocation or traffic sharing between different service-classes and b) forecasting of performance measures [1], [2].

We consider a single-link pure loss system of capacity $C$ bandwidth units (b.u.) that accommodates $K$ service-classes with different bandwidth requirements. Calls of service-class $k$ $(k = 1, \dots, K)$ arrive to the system according to a Poisson process and compete for the available link bandwidth under the threshold (TH) policy: A call of service-class $k$ is accepted in the system when its required bandwidth, $b_k$, is less than the available link bandwidth, while the number $n_k$ of in-service calls of service-class $k$ does not exceed a pre-defined threshold parameter, after its acceptance. The importance of the TH policy is twofold: i) It can be used to analyze a multirate access tree network which accommodates calls of $K$ different service-classes (see details in section II). This network consists of $K$ access links of capacity $C_k$ $(k = 1, \dots, K)$ b.u. and a common link of capacity $C$ b.u (Fig. 1), ii) the TH policy can be used to provide service-class differentiation in terms of CBP, revenue rates etc. Such a service differentiation can be achieved with a proper selection of the threshold parameters. A study on the optimal selection of the threshold parameters can be found in [3]. If calls of all service-classes do not have threshold parameters then the classical Erlang Multirate Loss Model (EMLM) results [4], [5].

In the EMLM, calls of each service-class compete for the available link bandwidth under the complete sharing policy. Calls are blocked and lost only if their required bandwidth is less than the available link bandwidth. Otherwise, they remain in the system for a generally distributed service time. To calculate the link occupancy distribution, Kaufman [4] and Roberts [5] proposed an accurate recursive formula which simplifies the CBP determination. This formula, known as Kaufman-Roberts formula, resulted in a large amount of extensions of the EMLM (e.g. [6]–[15]).

Tsang and Ross proposed in [16] an accurate recursive formula (named, herein, the Tsang/Ross recursion) for calculating CBP in a multirate access tree network by exploiting the fact that the steady state distribution of the number of calls in the system has a product form solution. In this paper, we adopt this recursion as a springboard to the calculation of CBP derivatives with respect to offered traffic-load. This calculation enables the study of the interaction between different service-classes that share the same system. Having determined the CBP derivatives with respect to offered traffic-load, we can approximate CBP for small variations of offered traffic-load. Such an approximation is helpful since the Tsang/Ross recursion has a worst case complexity of O$(K^m C)$ with $m = \max\{C/C_k, k = 1, \dots, K\}$, which is significant in large access tree networks and makes the approximate CBP determination through derivatives essential.

The remainder of this paper is as follows: In section II, we review the model of [16]. In section III, we propose an algorithm for the calculation of CBP derivatives with respect to offered traffic-load. In section IV, we present an approximate CBP formula based on the corresponding CBP derivatives. In section V, we present numerical examples for evaluation. We conclude in section VI.

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## II. CBP IN A TREE NETWORK

Consider a tree network that accommodates calls of $K$ service-classes. The network consists of $K$ access links of capacity $C_k$ $(k = 1, \dots, K)$ b.u. and a common link of capacity $C$ b.u. A call of service class $k$ $(k = 1, \dots, K)$ follows a Poisson process with arrival rate $\lambda_k$ and requests $b_k$ b.u. simultaneously on the $k^{th}$ access link and the common link. Without loss of generality we assume that $C_k$ is a multiple integer of $b_k$. If these $b_k$ b.u. are available then the call remains in the system for an exponentially distributed service time with mean $\mu_k^{-1}$. Otherwise the call is blocked and lost. The total offered traffic-load of service-class $k$ calls is $\alpha_k = \lambda_k \mu_k^{-1}$ (Fig. 1). If we denote by $n_k$ the number of in-service calls of service-class $k$ in the steady state and the corresponding vector $\boldsymbol{n} = (n_1, n_2, \dots, n_k, \dots, n_K)$ then the steady state distribution, $P(\boldsymbol{n})$, is given by [17]:

$$P(\boldsymbol{n}) = G^{-1} \left( \prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!} \right) \qquad (1)$$

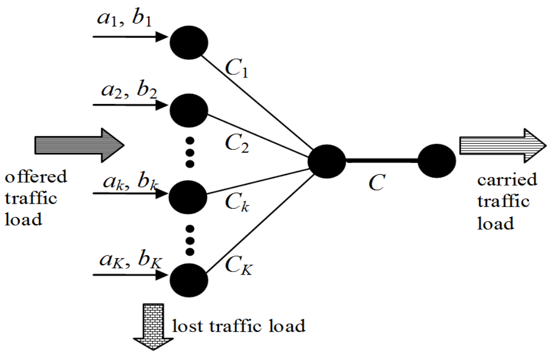


Fig. 1. A basic topology of a multirate access tree network.

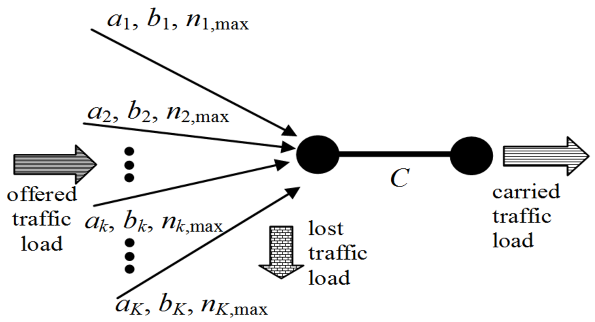


Fig. 2. A single link under the TH policy.

where $G$ is the normalization constant given by $G \equiv G(\boldsymbol{\Omega}) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}} \left( \prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!} \right)$ and $\boldsymbol{\Omega}$ is the state space of the system, $\boldsymbol{\Omega} = \{\boldsymbol{n} : 0 \le \boldsymbol{nb} \le C,\ n_k b_k \le C_k,\ k = 1, \ldots, K\}$, with $\boldsymbol{b} = (b_1, b_2, \ldots, b_k, \ldots, b_K)$ and $\boldsymbol{nb} = \sum_{k=1}^{K} n_k b_k$.

If $n_{k,\max} = C_k/b_k$ denotes the maximum number of service-class $k$ calls that the system can service then the state space $\boldsymbol{\Omega}$ can be rewritten as $\boldsymbol{\Omega} = \{\boldsymbol{n} : 0 \le \boldsymbol{nb} \le C,\ n_k \le n_{k,\max},\ k = 1, \ldots, K\}$. This form of $\boldsymbol{\Omega}$ represents a link of capacity $C$ that accommodates calls of $K$ service-classes which compete for the available bandwidth under the TH policy. By definition a policy is called a TH policy if there exists a set of positive integers $C_1, C_2, \ldots, C_k, \ldots, C_K$ such that a service-class $k$ call is accepted in the system, when in state $\boldsymbol{n}$, if and only if the new system state fulfils the relations $b_k(n_k + 1) \le C_k$ and $\boldsymbol{nb} + b_k \le C$ [2].

The previous discussion shows that the topology of Fig. 1 can be replaced by that of Fig. 2 and the analytical formulas obtained for the first topology can be used in the second one and vice versa. For presentation purposes we adopt the topology of Fig. 2 in the analysis that follows.

If we denote by $j$ the occupied link b.u. $(j = 0, \ldots, C)$ then the link occupancy distribution, $G(j)$, is given by the accurate Tsang/Ross recursion [16]:

$$G(j) = \begin{cases} 1 \text{ for } j = 0 \\ \frac{1}{j} \sum_{k=1}^{K} a_k b_k \left[ G(j-b_k) - T_k(j-b_k) \right] \text{ for } j = 1, \ldots, C \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

where $T_k(x)$ is the probability that $x$ b.u. are occupied while the number of service-class $k$ calls is $n_{k,\max}$ or:

$$T_k(x) := P[j = x,\ n_k = n_{k,\max}] \tag{3}$$

In eq. (3), the fact that $n_k = n_{k,\max}$ implies that: i) $j \ge n_{k,\max} b_k$ or $j \ge C_k$ and therefore $T_k(x) = 0$ for $x = 0, 1, \ldots, n_{k,\max} b_k - 1$ and ii) $T_k(x)$ is a blocking probability factor for service-class $k$ calls (since the number of in-service calls of service-class $k$ cannot exceed $n_{k,\max}$). As far as the CBP determination of service-class $k$ (denoted as $B_k$) is concerned, it is based on two groups of states: i) those where there is no available link bandwidth to accept a new service-class $k$ call; this happens when $C - b_k + 1 \le j \le C$ and ii) those where available link bandwidth exists, i.e. $j \le C - b_k$ but $n_k = n_{k,\max}$; the latter implies that $j \ge n_{k,\max} b_k$. The values of $B_k$ are given by:

$$B_k = \sum_{j=C-b_k+1}^{C} G^{-1} G(j) + \sum_{j=n_{k,\max} b_k}^{C-b_k} G^{-1} T_k(j) \tag{4}$$

where $G = \sum_{j=0}^{C} G(j)$ is the normalization constant.

To calculate the values of $T_k(j)$, when $j = n_{k,\max} b_k, \ldots, C - b_k$, let a subsystem having $F_k = C - b_k - n_{k,\max} b_k$ common b.u. and with service-class $k$ removed. For this subsystem, let $r_k(j)$, $j = 0, \ldots, F_k$, be analogous to $G(j)$ of eq. (2). To determine $r_k(j)$ we apply again eq. (2) to the subsystem assuming that service-class $k$ is removed. More precisely, the values of $r_k(j)$ are given by:

$$r_k(j) = \begin{cases} 1 \text{ for } j = 0 \\ \frac{1}{j} \sum_{\substack{i=1 \\ i \ne k}}^{K} a_i b_i \left[ r_k(j-b_i) - T_i(j-b_i) \right] \text{ for } j = 1, \ldots, F_k \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

Once the values of $r_k(j)$ are determined, we compute $T_k(j)$ for $j = n_{k,\max} b_k, \ldots, C - b_k$ according to the formula [16]:

$$T_k(j) = \frac{a_k^{n_{k,\max}}}{(n_{k,\max})!} r_k(j - n_{k,\max} b_k) \tag{6}$$

Note that if calls of all service-classes do not have threshold parameters then the EMLM results and the calculation of the link occupancy distribution, $G(j)$, is based on the classical Kaufman-Roberts formula [4], [5]:

$$G(j) = \begin{cases} 1 \text{ for } j = 0 \\ \frac{1}{j} \sum_{k=1}^{K} a_k b_k G(j - b_k) \text{ for } j = 1, \ldots, C \\ 0 \text{ otherwise} \end{cases} \tag{7}$$

The values of $B_k$, in the case of the EMLM, are given by:

$$B_k = \sum_{j=C-b_k+1}^{C} G^{-1} G(j) \tag{8}$$

where $G = \sum_{j=0}^{C} G(j)$ is the normalization constant.

## III. CBP Derivatives With Respect To Offered Traffic-Load

Based on eq. (4) the derivatives of $B_k$ with respect to offered traffic-load of service-class $i$ $(i = 1, \ldots, K)$ are given by:

$$\frac{\partial}{\partial a_i}B_k = \frac{\partial}{\partial a_i}\left(\sum_{j=C-b_k+1}^{C} G^{-1}G(j)\right) + \frac{\partial}{\partial a_i}\left(\sum_{j=n_{k,\max}*b_k}^{C-b_k} G^{-1}T_k(j)\right)$$
$$= \frac{\partial}{\partial a_i}\left(\sum_{j=C-b_k+1}^{C} G(j)\right) + \frac{\partial}{\partial a_i}\left(\sum_{j=n_{k,\max}*b_k}^{C-b_k} T_k(j)\right) - B_k\frac{\partial}{\partial a_i}G \quad (9)$$

where the determination of $\frac{\partial}{\partial a_i}B_k$ requires the calculation of $\frac{\partial}{\partial a_i}G(j)$ , $\frac{\partial}{\partial a_i}G$ and $\frac{\partial}{\partial a_i}T_k(j)$.

**a) Calculation of $\frac{\partial}{\partial a_i}G(j)$**

By definition $G(j)$ can be expressed by:

$$G(j) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j} P(n) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j}\prod_{k=1}^{K}\frac{a_k^{n_k}}{n_k!} \quad (10)$$

where $\boldsymbol{\Omega}_j = \{\boldsymbol{n} \in \boldsymbol{\Omega} : nb = j\}$.

Based on eq. (10), we determine $\frac{\partial}{\partial a_i}G(j)$ as:

$$\frac{\partial}{\partial a_i}\left(\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j}\prod_{k=1}^{K}\frac{a_k^{n_k}}{n_k!}\right) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j}\left(\prod_{k\neq i}^{K}\frac{a_k^{n_k}}{n_k!}\right)\frac{a_i^{n_i-1}}{(n_i-1)!}$$

which leads to the equation:

$$\frac{\partial}{\partial a_i}G(j) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j} P(\boldsymbol{n}_i^-) \quad (11)$$

where the vector $\boldsymbol{n}_i^- = (n_1, n_2, \ldots, n_i - 1, \ldots, n_K)$.

The values of $\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j} P(n_i^-)$ are expressed by:

$$\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j} P(\boldsymbol{n}_i^-) = G(j-b_i) - P(x = j-b_i, n_i = n_{i,\max}) \quad (12)$$

or by the following formula:

$$\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j} P(\boldsymbol{n}_i^-) = G(j-b_i) - T_i(j-b_i) \quad (13)$$

since $T_i(x = j - b_i) = P(x = j - b_i, n_i = n_{i,\max})$.

Based on eq. (13), we can write eq. (11) as follows:

$$\frac{\partial}{\partial a_i}G(j) = G(j-b_i) - T_i(j-b_i) \quad (14)$$

where the values of $G(j - b_i),\ T_i(j - b_i)$ have already been determined by eq. (2) and eq. (6), respectively.

**b) Calculation of $\frac{\partial}{\partial a_i}G$**

Since $G$ is the normalization constant we have:

$$\frac{\partial}{\partial a_i}G = \frac{\partial}{\partial a_i}\left(G(C) + ...G(b_i) + ... + G(0)\right) \quad (15)$$

Based on eq. (14), eq. (15) can be rewritten as:

$$\frac{\partial}{\partial a_i}G = G(C-b_i) - T_i(C-b_i) + G(C-b_i-1) + $$
$$-T_i(C-b_i-1) + ... + G(0) - T_i(0) \quad (16)$$

where the values of $G(j),\ T_i(j)$ have already been determined by eq. (2) and eq. (6), respectively.

**c) Calculation of $\frac{\partial}{\partial a_i}T_k(j)$**

Equation (6) shows that $T_k(j)$ is related to $r_k(j)$. Since $r_k(j)$ is analogous to $G(j)$, it is expressed by:

$$r_k(j) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j} P(n) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j}\prod_{\substack{i=1\\ i\neq k}}^{K}\frac{a_i^{n_i}}{n_i!} \quad (17)$$

where:
$\boldsymbol{\Omega}_j = \{\boldsymbol{n} \in \boldsymbol{\Omega} : \boldsymbol{nb} = j\}$,
$\boldsymbol{n}(n_1, n_2, \ldots, n_{k-1}, n_{k+1}, \ldots, n_K)$,
$\boldsymbol{b} = (b_1, b_2, \ldots, b_{k-1}, b_{k+1}, \ldots, b_K)$ and $\boldsymbol{nb} = \sum_{\substack{i=1\\ i\neq k}}^{K} n_i b_i$.

Based on eq. (17), we determine $\frac{\partial}{\partial a_i}r_k(j)$as:

$$\frac{\partial}{\partial a_i}\left(\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j}\prod_{\substack{i=1\\ i\neq k}}^{K}\frac{a_i^{n_i}}{n_i!}\right) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_j}\left(\prod_{\substack{l=1\\ l\neq i,\, i\neq k}}^{K}\frac{a_l^{n_l}}{n_l!}\right)\frac{a_i^{n_i-1}}{(n_i-1)!}$$

which leads to the equation: $\frac{\partial}{\partial a_i}r_k(j) = r_k(j - b_i)$ for $j = b_i, b_i+1, \ldots, F_k$ (18) where the values of $r_k(j-b_i)$ are given by eq. (5).

Based on eq. (6) and eq. (18), we calculate $\frac{\partial}{\partial a_i}T_k(j)$ as:

$$\frac{\partial}{\partial a_i}T_k(j) = \frac{a_k^{n_{k,\max}}}{(n_{k,\max})!}r_k(j - n_{k,\max}b_k - b_i) \quad (18)$$

for $i \neq k$, $j = n_{k,max}b_k, \ldots, C - b_k$ and $r_k(j) = 0$ when $j < 0$.

If $i = k$, from eq. (17) we have that $\frac{\partial}{\partial a_k}r_k(j) = 0$ since service-class $k$ is removed. Thus, eq. (6) becomes:

$$\frac{\partial}{\partial a_k}T_k(j) = \frac{\partial}{\partial a_k}\frac{a_k^{n_{k,\max}}}{(n_{k,\max})!}r_k(j - n_{k,\max}b_k)$$
$$= \frac{n_{k,\max}}{a_k}\frac{a_k^{n_{k,\max}}}{(n_{k,\max})!}r_k(j - n_{k,\max}b_k)$$
$$= \frac{n_{k,\max}}{a_k}T_k(j) \quad (19)$$

As a conclusion we have that:

$$\frac{\partial}{\partial a_i}T_k(j) = \begin{cases} \frac{a_k^{n_{k,\max}}}{(n_{k,\max})!}r_k(j - n_{k,\max}b_k - b_i), \text{ for } i \neq k \\ \\ \frac{n_{k,\max}}{a_k}T_k(j), \text{ for } i = k \end{cases} \quad (20)$$

To calculate the derivatives of $B_k$ with respect to offered traffic-load of service class $i$ we modify eq. (9), based on eq. (14), eq. (16) and eq. (20):

$$\frac{\partial}{\partial a_i}B_k = G(C-b_i) - T_i(C-b_i) + G(C-b_i-1) -$$
$$+ T_i(C-b_i-1) + G(C-b_i-b_k+1) -$$
$$+ T_i(C-b_i-b_k+1) + \frac{\partial}{\partial a_i}\left(\sum_{j=n_{k,\max}b_k}^{C-b_k} T_k(j)\right) -$$
$$+ B_k\left(G(C-b_i) - T_i(C-b_i) + ... + G(0) - T_i(0)\right) \quad (21)$$

where the values of $\frac{\partial}{\partial a_i}T_k(j)$ are given by eq. (20).

The validity of eq. (21) can be examined by the "reciprocity relation" whereby $\frac{\partial}{\partial a_i}B_k = \frac{\partial}{\partial a_k}B_i$ [18], [19]. Such a relation holds since the model has a product form solution (PFS). In non-PFS models this relation does not hold [20]–[22].

Note that in the case of the EMLM, eq. (9) becomes (see e.g. [22]):

$$\frac{\partial}{\partial a_i}B_k = \frac{\partial}{\partial a_i}\left(\sum_{j=C-b_k+1}^{C} G(j)\right) - B_k\frac{\partial}{\partial a_i}G \qquad (22)$$

where $\frac{\partial}{\partial a_i}G(j) = G(j-b_i)$, $\frac{\partial}{\partial a_i}G = 1 - B_i$ and the values of $G(j)$'s and $B_i$ are given by eq. (7) and eq. (8), respectively.

## IV. An Approximate CBP Formula

Having determined $\frac{\partial}{\partial a_i}B_k$according to eq. (21) we adopt the following approximate CBP formula [20]:

$$B_k(a_1+\Delta a_1,\, a_2+\Delta a_2, ..., a_K+\Delta a_K) = \\ = B_k(a_1,\, a_2, ..., a_K) + \frac{\partial B_k}{\partial a_1}\Delta a_1 + \frac{\partial B_k}{\partial a_2}\Delta a_2 + ... + \frac{\partial B_k}{\partial a_K}\Delta a_K \qquad (23)$$

when small variations (either positive or negative) of offered traffic $\Delta\alpha_k$ are considered.

Note that the values of $B_k(a_1,\, a_2, ..., a_K)$ have already been determined by eq. (4). Equation (23) can in general solve some basic problems, including the following [20]:

1) *Discrete Event Simulation*

Suppose we want to forecast CBP in a system, but have a rough idea of the offered traffic-load $(a_1,\, a_2, ..., a_K)$. In that case we should simulate the system for the values $(a_1+\Delta a_1,\, a_2+\Delta a_2, ..., a_K+\Delta a_K)$ which requires $K+1$ simulation runs and considerable execution time. Alternatively, we can simulate the system for $(a_1,\, a_2, ..., a_K)$ and then use eq. (23) to approximate CBPs in the vicinity of $(a_1,\, a_2, ..., a_K)$.

2) *Approximate CBP calculation*

If we can analytically determine CBPs for a given set $(a_1,\, a_2, ..., a_K)$ then we may use eq. (23) to approximate CBPs for $(a_1+\Delta a_1,\, a_2+\Delta a_2, ..., a_K+\Delta a_K)$.

To summarize our algorithm for the approximate CBP calculation we present the following steps:

1) For a given set of $(a_1,\, a_2, ..., a_{\mathrm{K}})$ calculate $G(j)$'s according to eq. (2).

2) Determine CBP according to eq. (4).

3) Calculate $\frac{\partial}{\partial a_i}B_k$according to eq. (21).

4) Determine approximate CBP for small variations of offered traffic-load $\Delta\alpha_k$. according to eq. (23).

## V. Numerical Examples - Evaluation

We examine the validity of the proposed algorithm (sections III and IV), through numerical examples. Note that simulation results of the model are not presented herein because the Tsang/Ross recursion is accurate. Consider a link of capacity $C$ = 50 b.u. that accommodates calls of three service-classes with the following characteristics:

TABLE I
Exact And Approximate Call Blocking Probabilities

| Offered traffic load points | $B_1$ (exact) | $B_1$ (appr.) | $B_2$ (exact) | $B_2$ (appr.) | $B_3$ (exact) | $B_3$ (appr.) |
|---|---|---|---|---|---|---|
| 1 | 0.0021 | 0.0019 | 0.0955 | 0.0953 | 0.0393 | 0.0373 |
| 2 | 0.0023 | 0.0022 | 0.0955 | 0.0954 | 0.0449 | 0.0438 |
| 3 | 0.0026 | 0.0025 | 0.0956 | 0.0955 | 0.0508 | 0.0504 |
| 4 | 0.0029 | 0.0029 | 0.0956 | 0.0956 | 0.0570 | 0.0569 |
| 5 | 0.0032 | 0.0032 | 0.0957 | 0.0957 | 0.0635 | 0.0635 |
| 6 | 0.0036 | 0.0036 | 0.0958 | 0.0958 | 0.0701 | 0.0699 |
| 7 | 0.0040 | 0.0039 | 0.0959 | 0.0959 | 0.0769 | 0.0765 |
| 8 | 0.0044 | 0.0042 | 0.0960 | 0.0960 | 0.0840 | 0.0831 |
| 9 | 0.0048 | 0.0046 | 0.0961 | 0.0961 | 0.0911 | 0.0896 |

1st service-class: $\alpha_1 = 9$ erl, $b_1 = 1$ b.u, $n_{1,\max} = 18$ calls

2nd service-class: $\alpha_2 = 2$ erl, $b_2 = 4$ b.u., $n_{2,\max} = 4$ calls

3rd service-class: $\alpha_3 = 1$ erl, $b_3 = 7$ b.u., $n_{3,max} = 3$ calls

This example corresponds to an access tree network with three access links of capacity $C_1 = 18$ b.u., $C_2 = 16$ b.u. and $C_3 = 21$ b.u. and a common link of $C = 50$ b.u.

In Table I, we present the exact CBP (according to eq. (4) and the approximate CBP (according to eq. (23)) for all service-classes, assuming that $\alpha_1$ and $\alpha_3$ change while $\alpha_2$ remains constant. The initial traffic-loads given above correspond to Point 5 of the first column of Table I. Below or above Point 5, $\alpha_1$ and $\alpha_3$ decrease or increase in steps of 0.1 erl and 0.05 erl, respectively. So, Point 1 corresponds to $(\alpha_1,\ \alpha_2,\ \alpha_3) = (8.6, 2, 0.8)$ and Point 9 to $(\alpha_1,\ \alpha_2,\ \alpha_3) = (9.4, 2, 1.2)$. The CBP derivatives needed in eq. (23) in order to determine CBPs for all Points (1 to 4 and 6 to 9) are calculated according to the initial traffic characteristics of Point 5. Based on the results of Table I, the approximation of CBP is very good compared to the exact CBP. Note also that the fact that $b_1 < b_2 < b_3$, in this example, does not necessarily mean that $B_3 > B_2 > B_1$ since according to eq. (4) the CBP of a service-class $k$ depend also on the maximum number of service-class $k$ calls that the system can service.

## VI. Conclusion

In this paper, we first review an accurate and recursive model for the CBP calculation in multirate access tree networks. Second we determine the CBP derivatives with respect to offered traffic-load and third based on the derivatives, we propose an algorithm for approximate CBP calculation. As we show, the proposed algorithm provides absolutely satisfactory CBP results.

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