Outage Analysis and Power Allocation for Space-time Block Coded Two-way Relaying

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***Abstract*—In this paper, we consider the two-way relay channel (TWRC) over which communication is performed using space-time block codes (STBC) with physical-layer network coding (PNC) protocol. The three-node two-way relaying (TWR) model consists of two end nodes (sources) simultaneously communicating with each other over a relay node where each node is equipped with two antennas and uses Alamouti's STBC to provide diversity. We consider non-reciprocal channels where fading channel coefficients are independent for opposite directions of the same link. We first derive closed form expressions for the outage probability of the STBC-TWR system and uncoded TWRC assuming equal transmit power at each node and decode-and-forward (DF) strategy at the relay. We show that the STBC-TWR system significantly improves the uncoded TWRC outage performance. Then, to further improve the performance, we propose an optimal power allocation in TWR scheme employing Alamouti's STBC and PNC protocol. The optimal power allocation is obtained by maximizing the sum-rate under the total power constraint. We show that the optimal power allocation considerably improves the outage performance as well as the sum-rate compared to the equal-power case.**

***Index Terms*—MIMO systems, space-time coding, two-way relaying.**

I. Introduction

In wireless communication systems, the use of relayed transmission to combat fading and broaden the coverage area is known as a promising alternative to the direct transmission between source and destination. However, the capacity of the classical three-node communication scheme where a source transmits its information to the destination with the aid of a relay, suffers from a pre-log factor of 1/2 due to the half-duplex relay which can not transmit and receive at the same time slot and same frequency band [1]. Two-way relay channels (TWRC) where two end nodes bidirectionally communicate via a relay when a direct link between them is not available, have attracted great interest in recent years. As well as the achievable transmission rate in one direction still suffers from the per-log factor of 1/2, bidirectional connection between end nodes increases the sum-rate of the system [1]. Physical-Layer Network Coding (PNC) which embraces interference of electromagnetic waves to improve the system throughput instead of treating it as a nuisance to be avoided, requires only two time slots to exchange two symbols between two end nodes and is considered as the most efficient two-way relaying protocol since bidirectional transmission is realized in four and three time slots for traditional scheduling and straigthforward network coding schemes, respectively [2]. The two time slots in PNC consist of a multiple access (MAC) phase during which two symbols simultaneously transmitted from the end nodes arrive at the relay as added in the physical layer and the broadcast (BC) phase during which the relay send a network coded symbol chosen depending on the applied strategies as amplify-and-forward (AF), decode-and-forward (DF) or partial decode-and-forward (PDF). AF strategy is relatively simple since only the amplified replica of the received signal is transmitted from relay. However, in AF the noise is also amplified at the relay which causes degradation in system performance at low signal-to-noise (SNR) region. In DF, relay decodes the symbols arriving from the two source nodes and sends them by applying a PNC mapping rule, a symbol from which each source node determines the symbol coming from the other node by extracting its self symbol from its received signal. The decoding complexity and the complexity of the PNC mapping rule at relay dramaticaly increase with increasing signal constellation size. The PDF avoids the noise amplification of AF and the PNC mapping rule of DF while keeping in the BC phase the fading effect of the MAC phase and reveals as an intermediate solution between them [3].

On the other hand, space diversity is a well-known technique to overcome wireless channel impairments such as multipath fading [5]. However, the classical three-node communication model with one antenna at each node which is widely considered for TWRC, does not provide diversity especially due to the absence of a direct link between communicating end nodes. In this case, space diversity can be exploited in BC phase by incorporating more than one relay in TWR scheme, source nodes being still equipped by one antenna. Consequently, distributed space-time coding previously proposed in [6] for one-way communications, is adapted to TWRC in [3]. The main problem of this scheme performing PDF strategy at relays, is the increased complexity with increasing number of relays. Recently, in [4], a simple protocol is proposed to reduce the computational complexity at relay nodes for the PDF strategy given in [3]. In order to provide space diversity, space-time block codes (STBC) [5] are prefered in practical situations where systems with very low encoding and decoding complexities are desired. Alamouti's STBC [7] in TWRC was recently considered in [8] where closed form expressions for symbol error probability of the combined scheme are derived for binary phase shift-keying (BPSK) and quadrature PSK (QPSK) modulations. It is also shown in [8] that this scheme with two antennas at each node, provides space diversity in both MAC and BC phases and that the DF strategy applied

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at relay provides better performance compared to the AF and PDF strategies.

In this paper, to provide space diversity we introduce Alamouti's STBC to the two-way relay networks applying DF strategy at relay as in [8], however, we consider the outage probability which is another important performance measure characterizing the transmission robustness. We derive an analytical expression for the outage probability to evaluate the impact of the STBC on the TWR system outage performance, and then to further improve the system performance, we provide a solution for the optimal power allocation problem, to maximize the achievable sum-rate of the TWR system under the total power constraint, assuming that the ideal channel state information (CSI) is available at all nodes.

The rest of this paper is organized as follows. Section II describes the considered system model. Section III is destinated to the outage probability analysis. The power allocation problem is considered in Section IV. In Section V, simulation results and comparisons are presented. Section VI concludes the paper.

II. System Model

We consider the two-way relay network where two end nodes (sources N_1 and N_2) simultaneously communicate with each other through a relay (R), all nodes being equipped with two antennas. At N_1 and N_2, each data block of q bits is mapped to an M-ary phase-shift keying (M-PSK) symbol where $M = 2^q$. Symbol pairs (s_{11}, s_{12}) and (s_{21}, s_{22}) of N_1 and N_2, respectively, are simultaneously transmitted to the relay using Alamouti's STBC according to the antenna/time schedule given in Table I where A_{ij} denotes jth (j=1,2) antenna of node N_i ($i = 1, 2$), T_{1p}, ($p = 1, 2$) representing the two consecutive time intervals of the MAC phase. During BC phase relay processes the received signals from its two antennas by applying the DF strategy and transmits a network-coded symbol pair to the end nodes as indicated in Table II where A_{Rk} denotes kth (k=1,2) antenna of the relay, T_{2p}, ($p = 1, 2$) representing the two consecutive time intervals of the BC phase. In Tables I and II $(.)^*$ denotes the complex conjugate operation. The relay maps the decided M-PSK symbols from the two end nodes into an M-PSK symbol according to a network coding mapping rule satisfying the conditions given in [2], which are required to ensure the equivalence of network coding arithmetic and electromagnetic-wave interference arithmetic. In this paper, we consider the mapping rule given in Table III [10] for QPSK where symbols are selected from bit pairs as follows: $00 \rightarrow x_1 = (1/\sqrt{2}, 1/\sqrt{2})$, $10 \rightarrow x_2 = (1/\sqrt{2}, -1/\sqrt{2})$, $01 \rightarrow x_3 = (-1/\sqrt{2}, 1/\sqrt{2})$ and $11 \rightarrow x_4 = (-1/\sqrt{2}, -1/\sqrt{2})$.

The transmitted symbols are affected by Rayleigh fading channel coefficients which remain constant during the transmission of two Alamouti's STBC symbol pairs from sources to the relay and during the transmission of a network coded symbol pair from relay to the end nodes, and change independently for different transmitted symbol pairs. Fading channel coefficients of the links from sources to the relay and from relay to the sources are denoted as in Fig.1. Note that in

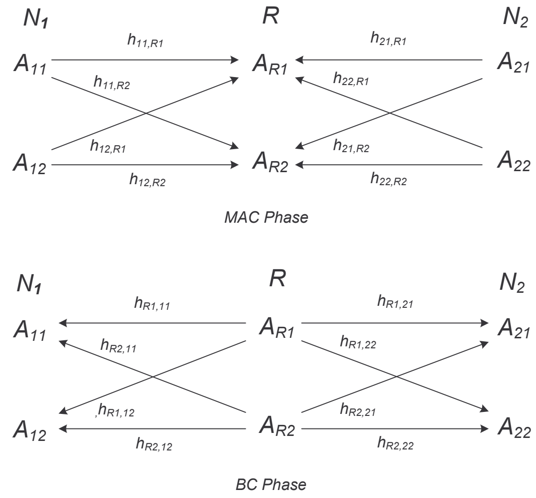

Fig. 1. Two-way relaying scheme employing Alamouti's STBC at each node.

this paper, we consider non-reciprocal channels where fading channel coefficients are independent for opposite directions of the same link. A half-duplex channel is considered where each antenna of each node cannot transmit and receive at the same time and same frequency. The complex additive white Gaussian noise (AWGN) is assumed having zero mean and variance 1.

TABLE I
Antenna/time schedule of the transmitted symbols during MAC phase

	N_1		N_2	
	A_{11}	A_{12}	A_{21}	A_{22}
T_{11}	s_{11}	s_{12}	s_{21}	s_{22}
T_{12}	$-s_{12}^*$	s_{11}^*	$-s_{22}^*$	s_{21}^*

TABLE II
Antenna/time schedule of the transmitted symbols during BC phase

	R	
	A_{R1}	A_{R2}
T_{21}	s_{R1}	s_{R2}
T_{22}	$-s_{R2}^*$	s_{R1}^*

TABLE III
PNC Mapping Rule for QPSK Modulation

$\hat{s}_{1j}$	x_1	x_1	x_1	x_1	x_2	x_2	x_2	x_2
$\hat{s}_{2j}$	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
s_{Rk}	x_1	x_2	x_3	x_4	x_3	x_1	x_4	x_2
$\hat{s}_{1j}$	x_3	x_3	x_3	x_3	x_4	x_4	x_4	x_4
$\hat{s}_{2j}$	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
s_{Rk}	x_2	x_4	x_1	x_3	x_4	x_3	x_2	x_1

Let the received signal at the kth antenna of the relay during the pth time slot of the MAC phase be denoted by r_{kp}. Then, omitting the transmit power factors of each node, the input-output relation of the multiple access channel can be written as

$$\mathbf{r} = H_M\mathbf{s} + \mathbf{n}, \tag{1}$$

where

$$\mathbf{r} = (r_{11}, r_{12}^*, r_{21}, r_{22}^*)^T$$

and

$$\mathbf{s} = (s_{11}, s_{12}, s_{21}, s_{22})^T$$

denote the received and transmitted signal vectors, respectively, and

$$\mathbf{n} = (n_{11}, n_{12}^*, n_{21}, n_{22}^*)^T$$

is the AWGN vector at the relay, with $(.)^T$ denoting the transpose operation. In (1),

$$H_M = \begin{bmatrix} h_{11,R1} & h_{12,R1} & h_{21,R1} & h_{22,R1} \\ h_{12,R1}^* & -h_{11,R1}^* & h_{22,R1}^* & -h_{21,R1}^* \\ h_{11,R2} & h_{12,R2} & h_{21,R2} & h_{22,R2} \\ h_{12,R2}^* & -h_{11,R2}^* & h_{22,R2}^* & -h_{21,R2}^* \end{bmatrix}$$

represents the channel coefficients matrix with independent and identically distributed entries, $h_{ij,Rk}$ being the fading coefficient of the channel from the jth antenna of the end node N_i to the kth antenna of the relay. If the relay applies maximum likelihood (ML) decoding to the received vector $\mathbf{r}$ to determine the decided signal vector

$$\hat{\mathbf{s}} = (\hat{s}_{11}, \hat{s}_{12}, \hat{s}_{21}, \hat{s}_{22})^T,$$

from

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} ||\mathbf{r} - H_M\mathbf{s}||^2, \tag{2}$$

the decoding complexity becomes equal to M^4 for $M-$PSK modulation due to the deterioration of the orthogonality at the relay in the MAC phase. Instead the relay can apply the conditional ML decoding previously proposed in [9] for decoding high-rate STBCs. When the relay applies conditional ML decoding, instead of computing M^4 ML metrics for all possible values of the symbol quadruple $(s_{11}, s_{12}, s_{21}, s_{22})$, it computes ML metrics for the pair (s_{11}, s_{12}) for each possible realization of the pair (s_{21}, s_{22}). When the pair (s_{21}, s_{22}) is fixed, its effect on the received vector $\mathbf{r}$ can be extracted as follows

$$\mathbf{z} = \mathbf{r} - H_{M,2}\mathbf{s}_2, \tag{3}$$

where $\mathbf{s}_2 = (s_{21}, s_{22})^T$, $H_{M,2}$ is the 4×2 matrix including the third and fourth columns of H_M and

$$\mathbf{z} = (z_{11}, z_{12}^*, z_{21}, z_{22}^*)^T.$$

Then using (1), (3) can be rewritten as

$$\mathbf{z} = H_{M,1}\mathbf{s}_1 + \mathbf{n}, \tag{4}$$

where $\mathbf{s}_1 = (s_{11}, s_{12})^T$ and $H_{M,1}$ is the 4×2 matrix including the first and second columns of H_M. Therefore, orthogonality is restored for $\mathbf{s}_1$ and the combined signal vector

$$\tilde{\mathbf{s}}_1 = (\tilde{s}_{11}, \tilde{s}_{12})^T = H_{M,1}^h\mathbf{z} \tag{5}$$
$$= (|h_{11,R1}|^2 + |h_{12,R1}|^2 + |h_{11,R2}|^2 + |h_{12,R2}|^2)I\mathbf{s}_1 + \mathbf{n}_0$$

is obtained with full diversity where $(.)^h$ stands for Hermitian operation, I is the 2×2 unitary matrix and $\mathbf{n}_0 = H_{M,1}^h\mathbf{n}$ is the noise vector. $\tilde{\mathbf{s}}_1$ is then sent to the ML decoder which decides separately to $\hat{s}_{11}$ and $\hat{s}_{12}$ by minimizing the Euclidean distances as follows:

$$\hat{s}_{11} = \arg\min_{s_{11}} d^2(\tilde{s}_{11}, s_{11}), \tag{6}$$
$$\hat{s}_{12} = \arg\min_{s_{12}} d^2(\tilde{s}_{12}, s_{12}),$$

where minimizations are over the M-PSK constellation. Finally the quadruple providing the minimum metric is selected. This means that $2M$ metric calculations are needed to determine the pair (s_{11}, s_{12}) for each possible realization of the pair (s_{21}, s_{22}) and the total metric calculations reduce to $2M \times M^2 = 2M^3$. Note that, the pair (s_{21}, s_{22}) can be determined as well by exploiting orthogonality, for each possible realization of the pair (s_{11}, s_{12}).

During the BC phase, relay transmits the symbol vector $\mathbf{s}_R = (s_{R1}, s_{R2})^T$ determined from $\hat{\mathbf{s}}$ according to a PNC mapping rule. For QPSK, the PNC mapping rule is given in Table III. Denoting the received signal at the jth antenna of the end node N_i during the pth time slot of the BC phase by $y_{i,jp}$, the input-output relation of the BC channel from relay to N_i is given by

$$\mathbf{y}_i = H_B\mathbf{s}_R + \mathbf{n}_i, \tag{7}$$

where

$$\mathbf{y}_i = (y_{i,11}, y_{i,12}^*, y_{i,21}, y_{i,22}^*)^T$$

and

$$\mathbf{n}_i = (n_{i,11}, n_{i,12}^*, n_{i,21}, n_{i,22}^*)^T$$

are the received signal vector and the AWGN vector at the ith end node, respectively. In (7)

$$H_B = \begin{bmatrix} h_{R1,i1} & h_{R2,i1} \\ h_{R2,i1}^* & -h_{R1,i1}^* \\ h_{R1,i2} & h_{R2,i2} \\ h_{R2,i2}^* & -h_{R1,i2}^* \end{bmatrix}$$

is the channel coefficients matrix with independent and identically distributed entries, $h_{Rk,ij}$ being the fading coefficient of the channel from the kth antenna of the relay to the jth antenna of the end node N_i. Each end node applies the single symbol ML decoding thanks to the orthogonality of the Alamouti code for the BC phase, namely, obtains the combined signal vector

$$\tilde{s}_R = (\tilde{s}_{R1}, \tilde{s}_{R2})^T = H_B^h\mathbf{y}_i \tag{8}$$

and decides separately, in favor of

$$\hat{s}_{R1} = \arg\min_{S_{R1}} d^2(\tilde{s}_{R1}, s_{R1}), \tag{9}$$
$$\hat{s}_{R2} = \arg\min_{S_{R2}} d^2(\tilde{s}_{R2}, s_{R2}),$$

where minimizations are over the M-PSK constellation. Then from the decided and its own transmitted symbol pairs determines the symbol pair transmitted from the other end node using the PNC mapping rule given in Table III for QPSK modulation.

III. Outage Analysis

If the transmission rate R in [bits/s/Hz] exceeds the instantaneous capacity of the channel with probability P_{out}, called the outage probability, such a transmission is possible with probability $1-P_{out}$ [5]. In this section, we derive closed form expressions for the outage probabilities of the STBC-TWR system and the uncoded TWR reference system assuming that all channels are non-reciprocal.

Let R_{12} and R_{21} denote the achievable rate from node N_1 to node N_2 and from node N_2 to node N_1, respectively. Then,

$$R_{12} = \min\{R_{1,R1}, R_{1,R2}, R_{R,21}, R_{R,22}\} \tag{10}$$

and

$$R_{21} = \min\{R_{2,R1}, R_{2,R2}, R_{R,11}, R_{R,12}\}, \tag{11}$$

where

$$R_{i,Rk} = \log_2\left[1 + (|h_{i1,Rk}|^2 + |h_{i2,Rk}|^2)\frac{P_i}{2}\right] \tag{12}$$

and

$$R_{R,ij} = \log_2\left[1 + (|h_{R1,ij}|^2 + |h_{R2,ij}|^2)\frac{P_R}{2}\right]. \tag{13}$$

In (12), $R_{i,Rk}$ denotes the achievable rate for the channel from the ith end node to the kth antenna of the relay and P_i is the transmit power of the ith end node. In (13), $R_{R,ij}$ stands for the achievable rate for the channel from relay to the jth antenna of the ith end node and P_R is the transmit power of the relay. Since each M-PSK symbol is sent from both antennas of each node, we assume that the transmit power of each node is equally shared by its two antennas. The achievable rate is bounded by

$$R \leq \frac{1}{2}\min\{R_{12}, R_{21}\} \tag{14}$$

and the outage probability is given by

$$P_{out} = 1 - \Pr\left[R \leq \frac{1}{2}\min\{R_{12}, R_{21}\}\right]. \tag{15}$$

In (14), the factor 1/2 is due to two channel uses required to transmit one symbol from node N_1 to node N_2 and vice versa. Then the achievable rate region [11] can be expressed as

$$\begin{aligned} R(h_0) = \{(&X_{1,R1}, X_{1,R2}, X_{R,21}, X_{R,22}, \\ &X_{2,R1}, X_{2,R2}, X_{R,11}, X_{R,12}) : \\ &\min(X_{1,R1}, X_{1,R2}, X_{R,21}, X_{R,22}, \\ &X_{2,R1}, X_{2,R2}, X_{R,11}, X_{R,12}) \geq h_0\}, \end{aligned} \tag{16}$$

where

$$h_0 = \frac{2^{2R}-1}{(P/2)}, \tag{17}$$

$P = P_1 = P_2 = P_R$ for equal power at each node and

$$\begin{aligned} X_{1,R1} &= X_1 = |h_{11,R1}|^2 + |h_{12,R1}|^2 \\ X_{1,R2} &= X_2 = |h_{11,R2}|^2 + |h_{12,R2}|^2 \\ X_{R,21} &= X_3 = |h_{R1,21}|^2 + |h_{R2,21}|^2 \\ X_{R,22} &= X_4 = |h_{R1,22}|^2 + |h_{R2,22}|^2 \\ X_{2,R1} &= X_5 = |h_{21,R1}|^2 + |h_{22,R1}|^2 \\ X_{2,R2} &= X_6 = |h_{21,R2}|^2 + |h_{22,R2}|^2 \\ X_{R,11} &= X_7 = |h_{R1,11}|^2 + |h_{R2,11}|^2 \\ X_{R,12} &= X_8 = |h_{R1,12}|^2 + |h_{R2,12}|^2. \end{aligned}$$

X_is are central chi-squared distributed i.i.d. random variables with degree of freedom $n = 4$ and p.d.f. given by

$$f(x_i) = \lambda^2 x_i e^{-\lambda x_i}, \tag{18}$$

where $\lambda = 1/2\sigma^2$ and σ^2 is the variance of the zero mean Gaussian distributed real and imaginary components of the fading channel coefficients. A one-directional linear network geometry is considered where the normalized distances between the two end nodes and the distance from N_1 to the relay are taken as equal to 1 and d, respectively. The channel fading coefficients between antennas of the node N_1 and those of the relay, and between the antennas of the node N_2 and those of the relay are modeled as $\frac{\eta_1}{d^{\alpha/2}}$ and $\frac{\eta_2}{(1-d)^{\alpha/2}}$, respectively, where η_1 and η_2 are circular symmetric complex Gaussian distributed random variables with zero-mean and variance 1, independently varying for the two directions of the corresponding link. The path loss component α is set to 3. The following eight congruent subregions with equal probability for $1 \leq i, j \leq 8$ satisfy the condition in (16):

$$R_i(\lambda, h_0) = \{X_i > h_0 \cap X_j > X_i, (\forall j \neq i)\}. \tag{19}$$

The probability of the region $R(h_0) = R(\lambda, h_0)$ is calculated as

$$\begin{aligned} \Pr[R(\lambda, h_0)] &= 8\Pr[R_i(\lambda, h_0)] \\ &= 8\int_{R_i(\lambda,h_0)} f(x_1, ..., x_8)dx_1...dx_8 \\ &= 8\int_{h_0}^{\infty}\left(\int_{x_i}^{\infty}\lambda^2 x_j e^{-\lambda x_j}dx_j\right)^7 \lambda^2 x_i e^{-\lambda x_i}dx_i \\ &= (\lambda h_0 + 1)^8 e^{-8\lambda h_0} \end{aligned} \tag{20}$$

and using (17) in (20) the outage probability of the STBC-TWR system is finally found as

$$\begin{aligned} P_{out} &= 1 - \Pr[R(\lambda, h_0)] \\ &= 1 - \left(2\lambda\frac{2^{2R}-1}{P} + 1\right)^8 \exp\left\{-8\lambda\frac{2^{2R+1}-2}{P}\right\}. \end{aligned} \tag{21}$$

Let us consider now, the outage probability of the uncoded non-reciprocal TWR reference system which consists of two end nodes (sources) and a relay node, each having one antenna. During the MAC phase, the fading coefficients of the channels from source nodes N_1 and N_2 to the relay node are denoted by h_{1R} and h_{2R}, respectively. Similarly, the fading coefficients of the channels from relay to the end nodes N_1 and N_2 are represented by h_{R1} and h_{R2}, respectively. The achievable rates for each of these channels are given by

$$\begin{aligned} R_{1R} &= \log_2(1+P|h_{1R}|^2) \\ R_{2R} &= \log_2(1+P|h_{2R}|^2) \\ R_{R1} &= \log_2(1+P|h_{R1}|^2) \\ R_{R2} &= \log_2(1+P|h_{R2}|^2), \end{aligned} \tag{22}$$

assuming equal power P is allocated to each node. The achievable rate is bounded by (14) with

$$\begin{aligned} R_{12} &= \frac{1}{2}\min\{R_{1R}, R_{R2}\} \\ R_{21} &= \frac{1}{2}\min\{R_{2R}, R_{R1}\}, \end{aligned} \tag{23}$$

while the achievable rate region can be expressed as

$$\begin{aligned} R(h_0) = \{&|h_{1R}|^2, |h_{2R}|^2, |h_{R1}|^2, |h_{R2}|^2 : \\ &\min\{|h_{1R}|^2, |h_{2R}|^2, |h_{R1}|^2, |h_{R2}|^2\} > h_0\}, \end{aligned} \tag{24}$$

where

$$h_0 = \frac{2^{2R}-1}{P}. \tag{25}$$

The outage probability for the uncoded non-reciprocal TWR system is given by,

$$P_{out}^{(u)} = 1 - \Pr[\min\{|h_{1R}|^2, |h_{2R}|^2, |h_{R1}|^2, |h_{R2}|^2\} > h_0]. \tag{26}$$

Since the absolute squared values of these four fading coefficients are independently exponential distributed as $f(x) = \lambda e^{-\lambda x}$ where $\lambda = 1/2\sigma^2$, similar to (19) and (20), (26) is obtained as

$$\begin{aligned} P_{out}^{(u)} &= 1 - 4\int_{h_0}^{\infty}\left(\int_{x_i}^{\infty}\lambda e^{-\lambda x_j}dx_j\right)^3 \lambda e^{-\lambda x_i}dx_i \\ &= 1 - e^{-4\lambda h_0}. \end{aligned} \tag{27}$$

The theoretical outage probability curves for the STBC-TWR system and uncoded TWRC are depicted in Figure 2 for the case where the relay is in the middle of the two end nodes $(d = 0.5)$.

IV. Power Allocation

We consider in this section, the achievable sum-rate maximization problem for the STBC-TWR system by appropriate allocation of the transmit power to the network nodes under the total transmit power $(P_T = P_1 + P_2 + P_R)$ constraint. The achievable sum-rate of the STBC-TWR system is given by

$$\begin{aligned} R_{sum} &= R_{12} + R_{21} \\ &= \frac{1}{2}\min\{R_{1R}, R_{R2}\} + \frac{1}{2}\min\{R_{2R}, R_{R1}\}, \end{aligned} \tag{28}$$

where

$$\begin{aligned} R_{1R} &= \log_2\left(1 + \frac{P_1}{2}X_{1R}\right), \\ R_{R2} &= \log_2\left(1 + \frac{P_R}{2}X_{R2}\right), \\ R_{2R} &= \log_2\left(1 + \frac{P_2}{2}X_{2R}\right), \\ R_{R1} &= \log_2\left(1 + \frac{P_R}{2}X_{R1}\right) \end{aligned} \tag{29}$$

and

$$\begin{aligned} X_{1R} &= \min\{(|h_{11,R1}|^2+|h_{12,R1}|^2), (|h_{11,R2}|^2+|h_{12,R2}|^2)\}, \\ X_{R2} &= \min\{(|h_{R1,21}|^2+|h_{R2,21}|^2), (|h_{R1,22}|^2+|h_{R2,22}|^2)\}, \\ X_{2R} &= \min\{(|h_{21,R1}|^2+|h_{22,R1}|^2), (|h_{21,R2}|^2+|h_{22,R2}|^2)\}, \\ X_{R1} &= \min\{(|h_{R1,11}|^2+|h_{R2,11}|^2), (|h_{R1,12}|^2+|h_{R2,12}|^2)\}. \end{aligned}$$

As indicated in [12] for the uncoded TWRC case, a suboptimal solution to the power allocation problem can be provided by making $R_{1R} = R_{R2}$ and $R_{2R} = R_{R1}$ to eliminate the useless power consumption. This yields the following suboptimum solution for the Alamouti coded TWR system:

$$\begin{aligned} P_1^* &= \frac{X_{R2}X_{2R}}{X_{1R}X_{R1} + X_{R1}X_{R2} + X_{R2}X_{2R}}P_T, \\ P_2^* &= \frac{X_{1R}X_{R1}}{X_{1R}X_{R1} + X_{R1}X_{R2} + X_{R2}X_{2R}}P_T, \\ P_R^* &= \frac{X_{1R}X_{2R}}{X_{1R}X_{R1} + X_{R1}X_{R2} + X_{R2}X_{2R}}P_T. \end{aligned} \tag{30}$$

The achievable sum-rate can be further increased by allocating the power saved in one out of the two end nodes to the other end node and relay. For this purpose, we represent the powers for the three nodes as

$$\begin{aligned} P_1 &= P_1^* \pm \Delta P_1, \\ P_2 &= P_2^* \pm \Delta P_2, \\ P_R &= P_R^* \pm \Delta P_R \end{aligned} \tag{31}$$

and we consider only the case where $X_{1R} > X_{2R}$, the derivations for the reverse case being straightforward. In order to determine ΔP_1, ΔP_2 and ΔP_R, useless power consumption which does not increase R_{12} and R_{21} should be eliminated which means that the two achievable rates for at least one link pair should be the same. When $X_{1R} > X_{2R}$, the powers of N_1 and the relay are increased to enhance R_{1R} and R_{R2} to the same level for which the following equality should be satisfied:

$$\log_2\left(1+X_{1R}\frac{P_1^*+\Delta P_1}{2}\right) = \log_2\left(1+X_{R2}\frac{P_R^*+\Delta P_R}{2}\right). \tag{32}$$

From (29) and (32) we obtain,

$$\Delta P_R = \frac{X_{1R}}{X_{R2}}\Delta P_1. \tag{33}$$

Then, we can write,

$$\Delta P_2 = \Delta P_1 + \Delta P_R = \left(1 + \frac{X_{1R}}{X_{R2}}\right)\Delta P_1. \tag{34}$$

The gain in achievable sum-rate can be expressed as,

$$\begin{aligned} \Delta R_{sum} &= \Delta R_{12} - \Delta R_{21} \\ &= \left[\log_2\left(1+X_{1R}\frac{P_1^*+\Delta P_1}{2}\right) - \log_2\left(1+X_{1R}\frac{P_1^*}{2}\right)\right] \\ &- \left[\log_2\left(1+X_{2R}\frac{P_2^*}{2}\right) - \log_2\left(1+X_{2R}\frac{P_2^*-\Delta P_2}{2}\right)\right]. \end{aligned} \tag{35}$$

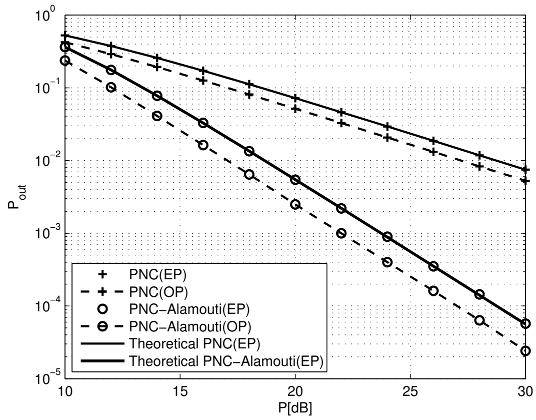

Fig. 2. Outage probabilities for uncoded and Alamouti coded TWR using PNC protocol (EP: Equal power, OP: Optimal Power)

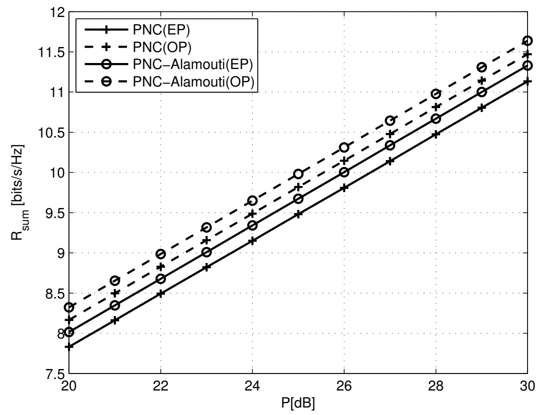

Fig. 3. Sum-rates for uncoded and Alamouti coded TWR using PNC protocol (EP: Equal Power, OP: Optimal Power).

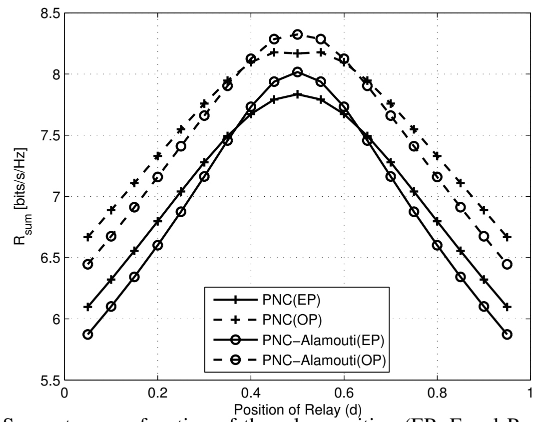

Fig. 4. Sum-rates as a function of the relay position (EP: Equal Power, OP: Optimal Power).

Using (34) in (35) and taking the derivative of ΔR_{sum} with respect to ΔP_1, we finally obtain,

$$\Delta P_1 = \frac{a_2 X_{1R} - a_1 a_3 X_{R2}}{2 a_3 X_{1R} X_{R2}}, \tag{36}$$

where

$$\begin{aligned} a_1 &= 1 + X_{1R} P_1^*, \\ a_2 &= 1 + X_{2R} P_2^*, \\ a_3 &= 1 + \frac{X_{1R}}{X_{R2}}. \end{aligned} \tag{37}$$

Therefore, the optimum power allocation for the Alamouti coded TWR system is

$$\begin{aligned} P_1^{opt} &= P_1^* + \Delta P_1, \\ P_2^{opt} &= P_2^* - \Delta P_2, \\ P_R^{opt} &= P_R^* + \Delta P_R, \end{aligned} \tag{38}$$

where ΔP_1, ΔP_2, and ΔP_R are as given in (36), (34) and (33), respectively.

V. Simulation Results

In this section, we evaluate the outage and sum rate performance of the STBC-TWR system and compare with that of the uncoded TWRC recently considered in [12]. The signal-to-noise ratio (SNR) at each node corresponds to P when equal power is transmitted from each node since the average noise power at each node is set to 1. When optimal power allocation is considered, the total power is constraint to $3P$. The QPSK modulation with $R = 2$ [bits/s/Hz] is considered and the transmit power of each node is equally divided to its two antennas for both equal power and optimal power allocation cases since each QPSK symbol is sent from both antennas of each node.

During the simulations, it was assumed that all channels were subject to the fading modeled as circular symmetric complex Gaussian distributed random variables with zero mean and variance 1, as indicated in Section III, and that the channel fading coefficients remain constant for each channel during the transmission of each Alamouti's symbol pair form sources in the MAC phase and from relay in the BC phase, and change independently from the transmission of one symbol pair to another. The channels were assumed non-reciprocal where fading channel coefficients are independently varying for opposite directions of a link. Moreover, it was assumed that the receivers have perfect knowledge of the channels terminating in themselves.

The outage probability curves obtained from theoretical analysis and through computer simulations for the Alamouti coded and uncoded TWRCs using the PNC protocol, are compared in Figure 2. The relay is located at the middle of the two end nodes $(d = 0.5)$. From Figure 2 we conclude that the simulation results match perfectly with the theoretical curves. Owing to the diversity provided by the Alamouti's code, the outage performance of the STBC-TWR system is significantly improved compared to the uncoded case. On the other hand, the application of optimum power allocation further improves both systems outage performance and provides approximately an SNR gain of 2 dB for both systems.

In Figure 3, the curves reflect the effect of the SNR on the achievable sum-rate for $d = 0.5$. The STBC-TWR system increases the achievable sum rate of about 0.35 [bits/s/Hz] compared to the uncoded TWRC, at the same SNR value. For the same achievable sum-rate the STBC-TWR system achieves an SNR gain of 0.5 dB, compared to the uncoded TWRC. The optimum power allocation provides an SNR gain of 1 dB at a fixed achievable sum-rate for both considered schemes.

The effect of the relay position on the achievable sum-rate is considered in Figure 4 for a P value of 20 dB. From these simulation curves we conclude that the STBC-TWR system outperforms the uncoded TWRC when $0.36 < d < 0.64$ for both equal power and optimum power allocation scenarios.

VI. Conclusion

In this paper, we have first derived closed form expressions for the outage probabilities of the Alamouti coded and uncoded TWR systems under the DF strategy at the relay and non-reciprocal fading channel assumptions, and we have evaluated the outage performance improvement compared to the uncoded case. Then, to maximize the achievable sum-rate of the Alamouti coded TWR system we have proposed an optimum power allocation under the total power constraint. The theoretical and simulation results show that the integration of a STBC into a TWR scheme significantly improves the outage performance and increases the achievable sum-rate.

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