Evaluation of Multicast Routing Algorithms with Fuzzy Sets
Maciej Piechowiak and Piotr Prokopowicz

Abstract—The paper presents a proposal of a new methodology that evaluates multicast routing algorithms in packet-switched networks with an application of fuzzy sets. Proposed multicriteria mechanism evaluate representative multicast routing algorithms: KPP, CSPT and MLRA (Multicast Routing Algorithm with Lagrange Relaxation) that minimize cost of paths between source and each destination node using Lagrange relaxation, and finally, minimize the total cost of multicast tree. A wide range of simulation research carried out by the authors, confirmed both the accuracy of new methodology and the effectiveness of the MLRA algorithm proposed by authors in earlier works.

Index Terms—multicast routing, fuzzy sets, research methodology.

I. INTRODUCTION

In common real-life problems often both parameters and data used in mathematical modelling are vague. For example if we do not have thermometer at the moment we think about temperature in terms: cold, warm, etc, and concerning that information we set up (often also in vague terms like: a bit more) heater (or ventilator). In mathematical modelling vagueness can be described by a fuzzy sets and numbers. Idea of the fuzzy sets was started in 1965 by Zadeh [1]. Fuzzy set theory has its well-known achievements in many branches of science and technology. Fuzzy concept have been introduced in order to model such vague terms, as observed values of some physical or economical terms, like pressure values or stock market rates, that can be inaccurate, can be noisy or can be difficult to measure with an appropriate precision because of technical reasons. In our daily life there are many cases that observations of objects in a population are fuzzy. Thus, the idea to implement fuzzy sets in evaluation of multicast routing algorithms were adopted.

Multicasting is a transmission method used in packet-switched networks for delivering mainly voice and multimedia data at the same time from one to many receivers. This technique requires efficient routing algorithms defining a tree with a minimum cost between the source node and the particular nodes representing the users. Such a solution prevents from duplication of the same packets in the links of the network. Routing of the sent data occurs only in those nodes of the network that lead directly to destination nodes.

The article discusses the effectiveness of the Multicast Lagrange Relaxation Algorithm (MLRA) and its comparison with most commonly used constrained heuristic algorithms: KPP (Kompella et al [2]) and CSPT (Crawford et al [3]).

II. NETWORK MODEL

Reliable evaluation of multicast routing algorithms require proper methodology of simulation research. The methodology should regard the way of generating network topology. Network generation method determines the placement of the nodes on the plane and the way of network nodes connection in a coherent structure. It also contains a method of generating link parameters (metrics). Each method has its own parameters, for example the Waxman model that constructs random graphs and defines parameters alpha and beta which affect the number of edges in the graph representing the network. Contrary to the random graph models, ad-hoc and sensor networks should be also take into consideration while evaluating routing algorithms. Ad-hoc networks were analyzed in many works, including [4], [5]. Mesh networks are under scrutiny of WING Project (Wireless Mesh Network for Next-Generation Internet) [6]–[8]. These publications provide detailed analysis on modeling topologies for ad-hoc networks as well as sensor networks, methods for controlling topologies, models of mobility of nodes in networks and routing protocols in wireless ad-hoc networks. Other part of research should also touch regular and quasi-regular network topologies. Arden and Lee [9] proposed using of chordal rings as the networking models. Further works ( [10], [11]) analyze the quality of chordal rings and improve their transmission abilities.

The network is represented by an undirected, connected graph $G = (V, E)$, where $V$ is a set of nodes, and $E$ is a set of links. With each link $e_{ij} \in E$ between nodes $i$ and $j$ two parameters are coupled: cost $c_{ij}$ and delay $d_{ij}$. The cost of a connection represents the usage of the link resources; $c_{ij}$ is then a function of the traffic volume in a given link and the capacity of the buffer needed for the traffic. A delay in the link is in turn the sum of the delays introduced by the propagation in a link, queuing and switching in the nodes of the network.

The multicast group is a set of nodes that are receivers of the group traffic (identification is carried out according to a unique $i$ address), $M = \{m_1, ..., m_m\} \subseteq V$. The node $s \in V$ is the...
source for the multicast group \( M \). Multicast tree \( T(s, M) \subseteq E \) is a tree rooted in the source node \( s \) that includes all members of the group \( M \) and is called Steiner tree.

The total cost of the Steiner tree \( T(s, M) \) can be defined as \( \sum_{t \in T(s, M)} c(t) \). The path \( p(s, m_i) \subseteq T(s, M) \) is a set of links between \( s \) and \( m_i \in M \). The cost of path \( p(s, m_i) \) can be expressed as: \( \sum_{p \in P(s, m_i)} c(p) \), where \( P(s, m_i) \) is a set of possible paths between \( s \) and \( m_i \). The delay is measured between the beginning and the end of the path as: \( \sum_{p \in P(s, m_i)} d(p) \). Thus, the maximum delay in the tree can be determined as: \( \max_{m_i \in M} [\sum_{p \in P(s, M)} d(p)] \).

Because of time complexity (the problem is proven to be \( \mathcal{NP} \)-hard) heuristic algorithms are most preferable for solving this problem.

### III. MULTICAST ROUTING ALGORITHMS

The implementation of multicasting requires solutions of many combinatorial problems accompanying the building of optimal transmission trees. In the optimization process it can be distinguished: MST – Minimum Steiner Tree, and the tree with the shortest paths between the source node and each of the destination nodes – SPT (Shortest Path Tree). Finding the MST, which is a \( \mathcal{NP} \)-complete problem, results in a structure with a minimum total cost [12]. The relevant literature provides a wide range of heuristics solving this problem in polynomial time [13]–[15]. From the point of view of the application in data transmission, the most commonly used is the KMB algorithm [13] and its modification – KPP algorithm [2] that reflect additional link parameter – delay.

During the first phase of the KPP, a complete graph is constructed whose all vertices are the source node \( s \) and the destination nodes \( m_i \in M \), while the edges represent the least cost paths connecting any two nodes \( a \) and \( b \) in the original graph \( G = (V, E) \), where \( a, b \in \{ M \cup s \} \). Then, the minimal spanning tree is determined in this graph taking the delay constraint \( \Delta \) into consideration, and then the edges of the obtained tree are inserted into the paths of the original graph \( G \). Any loops that appeared in this formed structure are removed with the help of the shortest path algorithm, for instance, by Dijkstra algorithm [16]. The computational complexity of the algorithm is \( O(|\Delta|V^3) \).

Other methods minimize the cost of each of the paths between the sender and each of the members of the multicast group by forming a tree from the paths having the least costs. In general, at first either the Dijkstra algorithm [16] or the Bellman-Ford algorithm [17] is used, then the branches of the tree that do not have destination nodes are pruned. Several routing algorithms have been proposed in the literature for this problem [3], [18]–[20].

In research studies the Constrained Shortest Path Tree (CSPT) is commonly used. It contains constrained shortest paths between the source and each of the destination node. The CSP problem (Constrained Shortest Path), also known as DCLC (Delay Constrained Least Cost), can be stated as the problem of minimizing: \( z^* = \min \sum_{i \in E} c_{ij}e_{ij}, \) subject to: \( \sum_{(i,j) \in E} d_{ij}e_{ij} \leq \Delta \), where: \( e_{ij} \in \{0,1\} \) [21].

Lagrange relaxation is a popular technique for calculating lower bounds and finding better solutions than popular CSP heuristics offer [21]. Held and Karp used this first to solve the traveling salesman problem [22]. It bases on modified cost function \( c_\lambda \) which is aggregated metric. Problem can be stated as follows:

\[
L(\lambda) = \min \sum_{(i,j) \in E} (c_{ij} + \lambda d_{ij})e_{ij} - \lambda \Delta,
\]

for certain \( \lambda: L(\lambda) = z^* \).

The proposed Path Lagrange Relaxation Algorithm (PLRA) refers to Jüttner et all idea [23]. Proposed algorithm relay on minimizing aggregated (modified) cost function: \( c_\lambda = c + \lambda d \). In each iteration of PLRA, the current value of \( \lambda \) is calculated, in order to increase the dominance of delay in the aggregated cost function, if the optimum solution of \( c_\lambda \) suits the delay requirements (\( \Delta \)).

The operation performed by the proposed Multicast Lagrange Relaxation Algorithm (MLRA) consists in determining the shortest path tree between source node \( s \) and each destination node \( m_i \), along which the maximum delay value (\( \Delta \)) cannot be exceeded. The paths determined one by one are added to the multicast tree. If there is at least one path that does not meet the requirements multicast tree cannot be constructed. Since the network structure created in that way may contain cycles, in order to avoid them Prim’s algorithm has been used.

In the last phase it removes leaves nodes with outdegree 1 that are not multicast nodes. On the basis of links entering the tree and their metrics (cost and delay) the total cost of the constrained tree is calculated.

### IV. INTRODUCTION TO FUZZY SETS

A fuzzy sets as a tool for the mathematical description of inaccuracies was proposed in 1965 by L. Zadeh. Definition of fuzzy set after Lofti Zadeh (1965).

**Definition 1:** Let \( X \) be a space of points (objects), with a generic element of \( X \) denoted by \( x \). Thus, \( X = x \). A fuzzy set \( A \subseteq X \) is characterised by a membership function \( f_A(x) \) which associates with each point in \( X \) a real number in the interval \([0,1]\), with the value of \( f_A(x) \) at \( x \) representing the "grade of membership" of \( x \) in \( A \).

\[
A = (x, f_A(x)) \forall x \in X,
\]

where \( f_A : X \rightarrow [0,1] \).

The fuzzy sets represents the imprecise terms which can be described in humans-like way. However, for practical use we need gather fuzzy sets into form of the linguistic variable. The linguistic variable [24] is a variable whose values are words from natural language. It mean if we want the parameter Distance to be linguistic variable we need describe its values in terms like fast, slow rather than numerical. Formal definition of a linguistic variable is presented below (after Lofti Zadeh [24]).

**Definition 2:** A linguistic variable is characterized by a quintuple \((V,T(V),U,G,M)\) in which \( V \) is the name of the variable; \( T(V) \) is the term set of \( V \), that is, the collection of its linguistic values; \( U \) is a universe of discourse; \( G \) is a syntactic rule which generates the terms in \( T(V) \); and \( M \) is
a *semantic rule* which associates with each linguistic value $X$ its meaning, $M(X)$, where $M(X)$ denotes a fuzzy subset of $U$. The meaning of a linguistic value $X$ is characterized by a compatibility function, $c : U \to [0, 1]$, which associates with each $u \in U$ its compatibility with $X$. The function of the semantic rule is to relate the compatibilities of the so-called *primary* terms in a composite linguistic value e.g. *fast* and *slow* in *not very fast and not very slow* to the compatibility of the composite value.

The concept of linguistic variable is a basic tool for linguistic model which can be transferred into fuzzy system.

The concept of fuzzy set was first formulated in 1965 [1]. Since that time it is constantly evolving and expanding. With the creation of the concept of linguistic variable and the fuzzy logic [24] it became possible to make a description of rules described linguistically in a purely mathematical way, that is, rules such as: IF there is little time and there is a long way ahead THEN you need to go much faster. A combination of such rules on a certain common context of action makes a fuzzy system. It provides a flexible and versatile tool for modeling all kinds of activities solely based on linguistic description that is available in each area, which human is able to describe in words, but not necessarily formally in a mathematical way.

Fuzzy systems can be divided into two main categories: models of Mamdani type and Takagi-Sugeno type (Takagi-Sugeno-Kang). Mamdani model is entirely based on an inaccurate linguistic description, and Takagi-Sugeno model combines the assumptions of imprecise model with full (precise) knowledge in the conclusions.

In addition to these major differences, there is the possibility to vary fuzzy systems in connection with the implementation of individual operations. The activity of fuzzy system can be represented as a sequence of consecutive stages. Over the years a number of methods was defined for each stage. There are many ways to implement the aggregation of complex premises. In the literature it is easy to find publications (e.g. [25]), the purpose of which is to compare and analyze the properties of aggregation operations. There are also different implications as well as actions replacing the implications called the implication operators [26]. There is also a number of defuzzification methods, properties of which are the subject of analyzes and comparisons. A good review of the level of knowledge of the early 21st century on the fuzzy systems is the publication [27].

Using the linguistic model is one of the methods for creating a fuzzy system. Main advantage is its availability. We need only a verbal description of the modeled situation. It is possible to create the mathematical model of expected relations by using the fuzzy system. It have a special significance in case of lack of precise mathematical models (also when achieving the precise model is particularly expensive, use of fuzzy system idea can be good proposal).

Such a situation occur in evaluation of multicast routing algorithms. We know what values are desired for the particular properties, but there are no guidelines how combine them together to get an assessment for the whole result. In this paper the fuzzy system is proposed as the multicriterial evaluation mechanism.

V. LINGUISTIC AND FUZZY MODEL

Authors describe preliminary analysis and the beginning of the research on applying fuzzy systems to the problem of multicriterial assessment of the multicast algorithms. Therefore the proposed linguistic model is exceptionally simple. Its relying on vital statistics of individual features concerning the assessment of the multicast network, so its complexity is minimized.

**Linguistic model assumptions:**

- **two input variables:**
  - *total cost of multicast tree* (TCMT) – Fig. 1a,
  - *average cost of path* between source and each destination node (ACP) – Fig. 1a.
- **one output variable:** *quality of multicast tree* (QMT) – Fig. 1b,
- **input variables** have two values: SMALL, HIGH,
- **output variable** have three values: SMALL, AVERAGE, HIGH,
- the domain of variables is normalized to interval $[0, 1]$.

**Parameters of fuzzy system:**

- **type of system:** Mamdani,
- **fuzzyfication method:** singleton,
- **aggregation operator for premise parts of fuzzy rules:** min,
- **implication operator in approximate reasoning:** min,
TABLE I
RESULTS OF MULTICRITERIA FUZZY EVALUATION FOR MULTICAST ALGORITHMS

<table>
<thead>
<tr>
<th>Parameter $\Delta$ = 2000</th>
<th>KPP</th>
<th>MLRA</th>
<th>CSPT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TCMT</td>
<td>ACP</td>
<td>Evaluation</td>
</tr>
<tr>
<td>Avg. val.</td>
<td>9746.3</td>
<td>1429.1</td>
<td>0.6137</td>
</tr>
<tr>
<td>Max. val.</td>
<td>14659</td>
<td>3733</td>
<td>0.7981</td>
</tr>
<tr>
<td>Min. val.</td>
<td>5837</td>
<td>572</td>
<td>0.4574</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter $\Delta$ = 2500</th>
<th>KPP</th>
<th>MLRA</th>
<th>CSPT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TCMT</td>
<td>ACP</td>
<td>Evaluation</td>
</tr>
<tr>
<td>Avg. val.</td>
<td>9789.4</td>
<td>1482.1</td>
<td>0.6208</td>
</tr>
<tr>
<td>Max. val.</td>
<td>16050</td>
<td>4755</td>
<td>0.8342</td>
</tr>
<tr>
<td>Minimal value</td>
<td>5374</td>
<td>558</td>
<td>0.4482</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter $\Delta$ = 3000</th>
<th>KPP</th>
<th>MLRA</th>
<th>CSPT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TCMT</td>
<td>ACP</td>
<td>Evaluation</td>
</tr>
<tr>
<td>Avg. val.</td>
<td>9692.9</td>
<td>1472.1</td>
<td>0.6028</td>
</tr>
<tr>
<td>Max. val.</td>
<td>15384</td>
<td>3809</td>
<td>0.7647</td>
</tr>
<tr>
<td>Min. val.</td>
<td>5116</td>
<td>462</td>
<td>0.4558</td>
</tr>
</tbody>
</table>

- aggregation method for fuzzy conclusions from rules: $\max$, defuzzification method: center of gravity.

Basics assumptions used to determine base of linguistic rules:
- total cost of multicast tree – the smaller, the better,
- average cost of paths between source and each destination node – the smaller, the better.

The principles presented earlier let constitute a set of four rules used in the fuzzy system:
- IF TCMT is SMALL AND ACP is SMALL THEN QMT is HIGH,
- IF TCMT is SMALL AND ACP is HIGH THEN QMT is MEDIUM,
- IF TCMT is HIGH AND ACP is SMALL THEN QMT is MEDIUM,
- IF TCMT is HIGH AND ACP is HIGH THEN QMT is SMALL.

In the further part of this article such fuzzy system will be named Multicriteria Fuzzy Evaluator (MFE).

VI. THE RESULTS OF EVALUATIONS

Due to a wide range of solutions presented in the literature of the subject, the following representative algorithms were chosen: KPP [2] and CSPT [3] algorithms and MLRA algorithm proposed by authors [28]. Such a set of algorithms includes solutions potentially most and least effective in terms of costs of constructed trees. The results were evaluated for three values of maximum delay ($\Delta$: 2000, 2500 and 3000).

In the first phase of experiment total cost of trees is examined (TCMT). The KPP algorithm constructs multicast trees with the minimum total cost among all results. CSPT algorithm creates multicast trees with highest costs on average.

In the second phase of the experiment average paths costs in multicast trees are examined (ACP). Analyzing average path’s cost (Tab. 1), it is observable that proposed MLRA algorithm construct trees with paths costs 40% lower on average in relation to the KPP algorithm in grid networks. MLRA algorithm achieve best results constructing trees in grid networks when maximum delay along path $\Delta$ is 2500.

In the last stage of experiment the Multicriteria Fuzzy Evaluator (MFE) was used in order to conduct independent evaluation for three algorithms and three $\Delta$ values. Results were normalized to a range between 0 and 1 (for each $\Delta$ value separately). Evaluating average values of total cost of multicast tree (TCMT) and average cost of path in tree (ACP) it is observable that results of MLRA algorithm are comparable with KPP algorithm (6.6%) for rigorous delay constraint ($\Delta$).

In this paper a flat random graph constructed graphs according to the Waxman method was used [12].

VII. CONCLUSIONS

The article presents a new technique for evaluation multicast algorithms. Research works were conducted for representative algorithms and shows that proposed earlier MLRA algorithm with Lagrange relaxation is more effective than popular CSPT algorithms. It was compared with representative routing algorithms for multicast connections emphasizing the quality of the network model.

It is difficult to find precise method for an evaluation of the multicast trees for large number of essential parameters. The method proposed in the article based on a simplified linguistic model which combine different estimations and join them into one estimation. This method allows for the effective implementation of multicriteria evaluations. Moreover, the results are fulfilling expectations and covers with previous (individual criterion) estimations for certain algorithms [28].

The proposed algorithm can be also successfully applied in large, dense networks with a high average node degree $D_{ave}$ [28]. The MLRA algorithm compared with CSPT and
KPP, achieves a good compromise between reasonable costs of trees and low time complexity.

REFERENCES


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